1. **Solution:** All the elementary final outcomes in the sample space are one of the following six types. The colors of the balls picked are going to be white,white or black,black or white,black,white or black,white,black or white,black,black or black,white,white. The random variable M, the number of balls remaining in the box, takes values 6 in the first two possibilities and 5 in the next four possibilities. The probability that the colors of balls is white, white (black,black) is $\frac{4*3}{8*7}$. The probability that the colors of balls is white,black, white (black,white,black) is $\frac{4*4*3}{8*7*6}$. The probability that the colors of balls is white,black,black (black,white,white) is $\frac{4*4*3}{8*7*6}$. Therefore

$$P(M=6) = 2 * \frac{4*3}{8*7} = \frac{3}{7}$$

$$P(M=5) = 4 * \frac{4*4*3}{8*7*6} = \frac{4}{7}.$$

$$P(M \le i) = \begin{cases} 0 & \text{if } i < 5 \\ \frac{4}{7} & \text{if } 5 \le i < 6 \\ 1 & \text{if } i \ge 6 \end{cases}$$

□

2. **Solution:** The distribution function F of the random variable W is a step function with jumps at $1, 2, 5$. Therefore W takes values $1, 2, 5$ with probabilities given by the jumps of F at $1, 2, 5$ respectively.

$$P(W=i) = \begin{cases} 0.40 & \text{if } i = 1 \\ 0.35 & \text{if } i = 2 \\ 0.25 & \text{if } i = 5 \end{cases}$$

gives the probabilities $P(W \le \frac{2}{5}) = 0$, $P(\frac{2}{5} \le W < \frac{5}{2}) = P(W=1) + P(W=2) = 0.75$ and the distribution function G of $2 - W$, which takes values $-3, 0, 1$ with probabilities $0.25, 0.35, 0.40$ respectively, is

$$G(x) = \begin{cases} 0 & \text{if } x < -3 \\ 0.25 & \text{if } -3 \le x < 0 \\ 0.60 & \text{if } 0 \le x < 1 \\ 1 & \text{if } 1 \le x \end{cases}$$

□

3. **Solution:** Let X be the index of the bag chosen at random and Y be the value of the coin chosen at random from the bag chosen.

$$\begin{aligned} P(X=1|Y=0.50) &= \frac{P(X=1, Y=0.50)}{P(Y=0.50)} \\ &= \frac{P(Y=0.50|X=1) * P(X=1)}{P(Y=0.50|X=1) * P(X=1) + P(Y=0.50|X=2) * P(X=2)} \\ &= \frac{\frac{1}{3} * \frac{1}{2}}{\frac{1}{3} * \frac{1}{2} + \frac{1}{7} * \frac{1}{2}} = 0.7 \end{aligned}$$

□

4. **Solution:** Given that Z is a Poisson random variable with parameter $\lambda > 0$. $P(Z = k) = e^{-\lambda}\frac{\lambda^k}{k!}$, $\forall k = 0, 1, 2....$

$$\begin{aligned}
\mathbb{E}(Z) &= \sum_{k=0}^{\infty} kP(Z = k) \\
&= \sum_{k=0}^{\infty} ke^{-\lambda}\frac{\lambda^k}{k!} \\
&= \lambda\sum_{k=1}^{\infty} e^{-\lambda}\frac{\lambda^{k-1}}{(k-1)!} = \lambda.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\mathbb{E}(Z(Z-1)) &= \sum_{k=0}^{\infty} k(k-1)P(Z = k) \\
&= \sum_{k=0}^{\infty} k(k-1)e^{-\lambda}\frac{\lambda^k}{k!} \\
&= \lambda^2\sum_{k=2}^{\infty} e^{-\lambda}\frac{\lambda^{k-2}}{(k-2)!} = \lambda^2.
\end{aligned}$$

Therefore

$$Var(Z) = \mathbb{E}(Z^2) - (\mathbb{E}(Z))^2 = \lambda.$$

By linearity of Expectation, $\mathbb{E}(V) = \mathbb{E}(2Z - 5) = 2\lambda - 5$ and $Var(V) = Var(2Z - 5) = Var(2Z) = 4 * Var(Z) = 4\lambda$.

□

5. **Solution:** $A_1 = 4X$ and $A_2 = 2(X + Y)$ where X, Y are the outcomes of two independent throws of a fair die. $X + Y$ takes values from 2 to 12.

   $P(X + Y = k) = \sum_{a=1}^{6} P(X = a)P(Y = k - a) = \frac{k-1}{36}$, if $k = 2, 3.., 7$ and $P(X + Y = k) = \frac{13-k}{36}$, if $k = 8, 9, 10.., 12$.

□

6. **Solution:** $C = \sum_{k=1}^{35} X_k$, where each X_k takes values $0, 1$. X_k is 1 if $k-$th student gets his own nametag, otherwise $X_k = O$. So, $P(X_k = 1) = \frac{1}{35}$.

$$\mathbb{E}(C) = \sum_{k=1}^{35} \mathbb{E}(X_k) = 1.$$

□

7. **Solution:**

$$
\begin{aligned}
1 - P(\bigcap_{i=1}^{n} A_i) &= P(\bigcup_{i=1}^{n} A_i^c) \\
&\leq \sum_{i=1}^{n} P(A_i^c) \\
&= n - \sum_{i=1}^{n} P(A_i).
\end{aligned}
$$

This gives the required inequality.

$\square$